Topic – Partial Differential Equations

Solve :– $z(xp-yq) = y^2-x^2$

Soln :– The given equation can be written as $zxp - zyq = y^2 - x^2$ — Eqn

The subsidiary equations are $\frac{dx}{zx} = \frac{dy}{-zy} = \frac{dz}{y^2-x^2}$

From the first two ratios, we have

$$\frac{dx}{x} = \frac{dy}{-y} \Rightarrow \int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\Rightarrow \log x = -\log y + \log c_1$$

$$\Rightarrow \log x + \log y = \log c_1$$

$$\Rightarrow \log(xy) = \log c_1$$

$$\Rightarrow xy = c_1 \quad \text{——} \quad (1)$$

Now using x, y, z as multipliers, we see that each ratio is

$$\frac{x\,dx + y\,dy + z\,dz}{zx^2 - zy^2 + z(y^2-x^2)} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$\Rightarrow x\,dx + y\,dy + z\,dz = 0$$

$$\Rightarrow \int x\,dx + \int y\,dy + \int z\,dz = 0$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = b$$

$$\Rightarrow x^2 + y^2 + z^2 = c_2 \text{ where } c_2 = 2b \quad - (2)$$

Hence the general solution is $\Phi(xy, x^2+y^2+z^2) = 0$